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277. Proposed by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

It is tacitly assumed in elementary geometry that as the number of sides of a regular polygon inscribed in a circle is increased, *in any manner*, that its perimeter has a fixed limit. Beginning with a square and then continually doubling the number of sides we get for the perimeter $2^{n+2}\sqrt{2-E^n(0)}$, where $E(x) \equiv \sqrt{2+x}$. Beginning with a hexagon we get $2^{m+1}3\sqrt{2-E^m(1)}$. The definition of the length of a circle assumes that these expressions have the same limit as $n \doteq \infty$ and $m \doteq \infty$. Prove it.

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Putting $\cos x = b$, we have $\sin \frac{1}{2}x = \sqrt{\frac{1}{2}(1-b)} = \frac{1}{2}\sqrt{2-2b}$, $\cos \frac{1}{2}x = \frac{1}{2}\sqrt{2+2b}$, $\sin \frac{1}{4}x = \frac{1}{2}\sqrt{2-\sqrt{2+2b}}$, $\cos \frac{1}{4}x = \frac{1}{2}\sqrt{2+\sqrt{2+2b}}$, $\sin \frac{1}{8}x = \frac{1}{2}\sqrt{2-\sqrt{2+\sqrt{2+2b}}}$, etc. Let the radius of a circle $= 1$, then we may represent the perimeter of a regular polygon by $2^{n+2}\sin \frac{1}{2^n}x$. For $n = \infty$, this assumes the indeterminate form $\infty \doteq 0$; but we may reduce it to $\frac{2^2 \sin (1/2^n)x}{1/2^n}$.

Differentiating numerator and denominator with reference to n , we get $2^2 x \cos(1/2^n)x$. For $n = \infty$, this becomes $2^2 x$. For $x = 90^\circ$, $2^2 \times 90 = 360^\circ$.

In regard to the hexagon, we have similarly, $\frac{3 \sin(1/2^n) \times 120^\circ}{1/2^n}$, which after differentiation as above, becomes $3 \times 120^\circ = 360^\circ$, so that the limit in both cases is 360° .

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$E^n(0) = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}};$$

$E^m(1) = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}$, the last term in the root being $\sqrt{3}$.

$$\text{Let } E^n(0) = E^m(1) = x. \quad \therefore \sqrt{2+x} = x, \text{ or } x=2.$$

$$\frac{2^{n+2}\sqrt{2-E^n(0)}}{2^{m+1}3\sqrt{2-E^m(1)}} = \frac{2^{n+2}\sqrt{2-x}}{2^{m+1}\sqrt{18-9x}} = \frac{2^{n+1}\sqrt{8-4x}}{2^{m+1}\sqrt{18-9x}}.$$

$$\frac{2^{n+1}}{2^{m+1}} \cdot \frac{\sqrt{8-4x}}{\sqrt{18-9x}} = \frac{\sqrt{8-4x}}{\sqrt{18-9x}}, \text{ when } n=m=\infty.$$

$$\text{If } \frac{\sqrt{8-4x}}{\sqrt{18-9x}} = 1, \quad 8-4x = 18-9x, \text{ or } x=2,$$

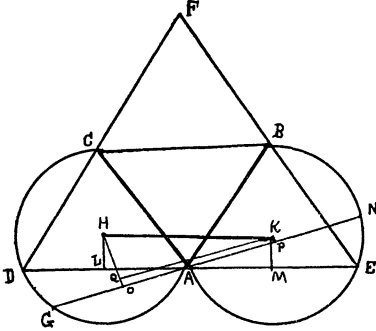
the same value as found above for x . Hence the expressions are equal when n and m are indefinitely increased.

279. Proposed by C. C. WENTWORTH, C. E., Roanoke, Va.

To construct geometrically the maximum equilateral triangle circumscribed about a given triangle.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Let ABC be the given triangle. On AC , AB describe segments to contain angles of 60° . Let H , K be the centers of these circles. Join HK and parallel to HK draw DAE . Join DC and EB , and produce DC , EB until they meet in F . Then FDE is the required maximum equilateral triangle.



$$\angle D = \angle E = \angle F = 60^\circ.$$

Draw any other line GN through A . Let fall the perpendiculars HL , KM , HO , KP , and draw KQ parallel to GN .

$$KQ = OP = \frac{1}{2}GN; HK = LM = \frac{1}{2}DE, HK > KQ. \therefore DE > GN.$$

$\therefore DE$ is the longest line that can be drawn through A .

$\therefore FDE$ is the maximum equilateral triangle required.

The same method will serve to describe the maximum triangle having any angles, about ABC .

Also solved by J. Scheffer.

MISCELLANEOUS.

155. Proposed by A. H. HOLMES, Brunswick, Maine.

There are two vessels, one containing a gallons of alcohol, the other b gallons of water. Suppose that c gallons are simultaneously taken from each and poured into the other, how many times must this be done so that there will be the same proportion of alcohol to water in each vessel?

I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let $F(x)$ represent the number of gallons in the first vessel after x operations, then the number of gallons in the first vessel after the next operation will be $F(x) - \frac{c}{a}F(x) + \frac{c}{b}[a - F(x)]$. $\therefore F(x+1) - [1 - (\frac{c}{a} + \frac{c}{b})]F(x) = \frac{ac}{b}$.

By the calculus of finite differences, we get

$$F(x) = \frac{a^2}{a+b} + C[1 - (\frac{c}{a} + \frac{c}{b})]^x.$$

Since for $x=0$, $F(x)=a$, we get $C = \frac{ab}{a+b}$.

$$\therefore F(x) = \frac{a^2}{a+b} + \frac{ab}{a+b}[1 - (\frac{c}{a} + \frac{c}{b})]^x.$$